STATISTICS

00011

Paper—III

Full Marks : 100

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

The paper is in **two** Parts. Attempt both the Parts.

PART—1

Answer *any* **five** questions

1. *(a)* A candidate applies for a job in two firms S and T. The probability of his being selected in firm S is 0·7 and being rejected in firm T is 0·5. The probability of at least one of his applications being rejected is 0·6. What is the probability that he will be selected in one of the firms? 4

   (b) From a box containing 3 gold and 5 silver coins, 4 coins are transferred into an empty box. Then, from this box, a coin is drawn and is found to be golden. What is the probability that out of the 4 coins transferred, 3 were gold and 1 was silver? Which theorem of probability is used in solving the above problem? 4+1=5

   (c) Define probability generating function (p.g.f.) of a random variable. 1

2. *(a)* If X is a random variable with probability density function (p.d.f.) $f(x) = 4x^3$, where $0 \le x \le 1$, find—

 (i) $P\left[\frac{1}{4} \le X \le \frac{1}{2}\right]$

 (ii) median of X

 (iii) Var (X) 3

(b) With usual notations, find p for a binomial variate X, if $n = 6$ and $9P(X = 4) = P(X = 2)$. 3

(c) If X and Y are independent Poisson variates, show that the conditional distribution of X given $X + Y$ is binomial. 4

3. *(a)* Draw the sketch of a normal probability curve and show the area property of the normal distribution in it. 3

(b) Two independent random variables X and Y are both normally distributed with means 1 and 2 and standard deviations 3 and 4 respectively. If $Z = X - Y$, write down the probability density function of Z. Also state the mean and variance of the distribution of Z. 3

(c) Prove that for the normal distribution, the quartile deviation, the mean deviation and the standard deviation are in the ratio $10 : 12 : 15$. 4

(Continued)

4. *(a)* Point out the difference between one-tailed and two-tailed tests in testing of hypothesis. Explain the concept of critical value with illustrations and write down the 5% and 1% critical values for both one-tailed and two-tailed tests when the sample size $n > 30$. 5

(b) If T is an unbiased estimator for θ with $V(T) > 0$, show that T^2 is a biased estimator for θ^2. 2

(c) Obtain the maximum likelihood estimate of θ in

$$f(x, \theta) = (1 + \theta)x^\theta, \ 0 < x < 1$$

based on an independent sample of size n. Examine whether the estimate is sufficient for θ. 3

5. *(a)* When are two attributes said to be completely associated? 800 candidates of both sexes appeared in an examination. The boys outnumbered the girls by 15% of the total. The number of candidates who were successful in the examination exceeds the number of failures by 480. It was found that equal number of boys and girls failed in the examination. Prepare a 2×2 table and find the coefficient of association. Comment on your results. 1+4=5

(Turn Over)

(4)

(b) Solve the following linear programming problem : 5

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$

subject to the constraints

$$3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

6. (a) Reduce the following matrix to canonical (normal) form and hence find its rank : 5

$$A = \begin{pmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

(b) Using Simpson's $\frac{3}{8}$th rule, obtain the approximate value of

$$\int_0^6 \frac{dx}{(1+x^2)}$$ 5

(5)

PART—2

Answer *any* **five** questions

7. (a) What do you mean by analysis of variance (ANOVA)? Discuss the effects of violations of the assumptions made in ANOVA. 5

(b) Explain the concept of Latin square design (LSD). Elaborate on the procedure of framing its layout, mentioning about the application of the principles of design of experiments. Write down the model used to define an LSD, explaining the terms used. 5

8. (a) Write a short note on sampling biases. 3

(b) Describe precisely the procedure for drawing a systematic sample of n units from a population of N units, when—

(i) $N = nk$, k is any integer;

(ii) $N \neq nk$. 4

(c) Discuss the similarities of the methods of stratified random sampling and randomised block design. 3

9. (a) Define and describe the measures which are used for comparing (i) the mortality situations and (ii) the fertility situations in two different regions. 4

(b) Interpret the results NRR = 1 and NRR > 1. 2

(c) Distinguish between a complete and an abridged life table. Mention two uses of a life table. 4

10. (a) From the following data, calculate price index numbers for the year 2005 with 1995 as base by using—

(i) Laspeyres' formula

(ii) Paasche's formula

(iii) Marshall-Edgeworth formula

(iv) Fisher's formula :

Commodities	1995		2005	
	Price	Quantity	Price	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Also verify that Marshall-Edgeworth index number is a good approximation to Fisher's index number. 4+1=5

(b) (i) What is meant by consumer price index number? State two methods of construction of this index number. 2+1=3

(ii) Given that consumer price index number (CPIN) for the current year is 325. It is also given that Mr. X, who was getting a salary of ₹ 24,000 p.m. in the base year, is presently getting ₹ 43,000 p.m. in the current year. Considering the value of CPIN, state how much Mr. X should have received as extra allowance to maintain his former standard of living. 2

11. (a) Describe briefly the components of a time series. 4

(b) Given below are the figures of production (in thousand tonnes) of a sugar factory :

Year (t) : 2013 2014 2015 2016 2017 2018 2019

Production (y_t) : 77 88 94 85 91 98 90

(i) Fit a straight line by the method of least squares and obtain the trend values.

(ii) Find the monthly increase in production. 3+1=4

(c) Mention two merits and two demerits of the method of moving averages for determining trend in a time series. 2

12. *(a)* Explain the basic principles underlying the construction of control charts bringing out the difference between natural tolerance limits and specification limits.   4

(b) Two variables X and Y have regression equations

$$3x + 2y - 26 = 0 \quad \text{and} \quad 6x + y - 31 = 0$$

Find the mean value and the coefficient of correlation between X and Y.   3

(c) How can scatter diagram be used to obtain an idea of the extent and nature of correlation between two variables?   3

★ ★ ★